MTH 605: Topology I

Homework II

(Due 25/08)

1. Show that for a function $f : \mathbb{R} \to \mathbb{R}$, the $\epsilon - \delta$ definition of countinuity is equivalent to the open set definition.

2. An indexed family of sets $\{A_\alpha\}$ is said to be *locally finite* if each point x of X has a neighborhood that intersects A_α for only finitely many values of α. Let $\{A_\alpha\}$ be a locally finite collection of closed subsets of X such that $X = \cup A_\alpha$. Show that if $f|_{A_\alpha}$ is continuous for each α, then f is continuous.

3. If (X, d) is a metric space, then the topology induced by d is the coarsest topology relative to which the function d is continuous.

4. Let $A \subset X$, and let $f : A \to Y$ be a continuous map of A into a Hausdorff space Y. Show that if f may be extended to a continuous function $g : \bar{A} \to Y$, then g is uniquely determined by f.

5. Prove that an uncountable product of $\mathbb{R}$ with itself is not metrizable.

6. Given $p \geq 1$, define
$$d(x, y) = \left[\sum_{i=1}^{n} |x_i - y_i|^p \right]^{1/p},$$
for $x = (x_1, \ldots, x_n), y = (y_1, \ldots, y_n) \in \mathbb{R}^n$. Show that d is a metric that induces the standard topology on $\mathbb{R}^n$.

7. Let $\mathbb{R}_0$ be the subset of $\mathbb{R}^\infty$ consisting of sequences in $\mathbb{R}$ that are eventually 0. Find the closure of $\mathbb{R}_0$ in $\mathbb{R}^\infty$ under the product and box topologies.

8. Define a map $h : \mathbb{R}^\infty \to \mathbb{R}^\infty$ that is linear in each coordinate. Is h continuous under the product and box topologies?